Readers' Forum

Comment on "Computation of Choked and Supersonic Turbomachinery Flows by a Modified Potential Method"

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THE authors of Ref. 1 have presented an interesting and useful method for computing internal flows with shock waves using a modified potential method. It is the purpose of this Note to add to the understanding of the limits of potential theory at transonic speeds by attempting to clarify some of the statements made in the subject paper, using asymptotic solutions derived for internal flows.

The statement that a potential solution may not exist at all in internal flows with a prescribed back pressure, and that when one does, the location of the shock wave is not unique, should be interpreted in the light of the order of the approximation one desires. Thus, as shown in Refs. 2 and 3 for steady flows and in Refs. 4-6 for unsteady flows, it is possible, in transonic internal flows, to locate the shock wave uniquely using a potential solution valid through third order. That is, if $|M^2-1|=0(\epsilon)$, say, then the (nondimensional) jump in entropy across the shock wave is $0(\epsilon^3)$. However, the Crocco equation shows that vorticity depends on the gradient in entropy normal to the streamlines, which is of still higher order, and so a potential exists at least through $0(\epsilon^3)$. It turns out that only second-order terms are necessary to locate the shock wave in steady and unsteady flow; thus, the shock is located by setting the back pressure to second order. This result may or may not be helpful in formulating numerical solutions to internal flows. In any event, it points out a reason for the difficulties found in determining the location of the shock wave by setting the back pressure in transonic flow; very small changes in back pressure can cause large-amplitude excursions in the shock position, and so truncation errors can be very important, as can the method of imposing the boundary condition at the duct exit.

The previous remarks also apply in interpreting the description of the solution envelopes given in Fig. 3 of Ref. 1. Thus, when a shock wave occurs in the diverging part of a channel, a jump takes place from the supersonic solution to a subsonic solution, which may be close to but is not the same as the one that passes through sonic velocity at the throat. In fact, for every shock position, there is of course a separate and unique subsonic solution to which the solution jumps, depending on the back pressure. Again, for weak shock waves it should be emphasized that although the jump in entropy is $0(\epsilon^3)$, the shock location may be found by setting back pressure using solutions valid to $0(\epsilon^2)$; the entropy jump need not be found in order to locate the shock wave. This does not mean that one cannot find the shock position using the jump entropy, of course, but only that it is not necessary to do so. As shown in Ref. 2, one finds the same result using either condition.

The general ideas developed in Ref. 2 have been used to find solutions for transonic flow through a lightly loaded cascade,[7] and a simple analytical expression for the unique incidence angle was given for the case of supersonic incoming velocity. Although only the flow ahead of the cascade was considered in detail for this case, the solutions for the flow regions between the blades should follow those found for channels closely enough that the results mentioned above concerning shock location would certainly hold there as well.

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Reply by Authors to T. C. Adamson Jr., J. Mace, and A. F. Messiter

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WE would like to thank Professors Adamson and Messiter and Dr. Mace for their interesting comments on our paper.[1] It is indeed useful for us, as well as others, to know about their publications listed in their comment.

In fact we completely agree with the remarks concerning the usefulness of the potential formulation and the methods to remedy its deficiencies. As stated in Ref. 2 of their com-

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ments, a correction to the potential solution is needed; otherwise the Rankine-Hugoniot shock jump conditions cannot be satisfied. Such a correction procedure allows the determination of the shock position in terms of the exit pressure.

It is obvious that their work is heavily dependent on asymptotic developments, while ours is purely numerical. Nevertheless, there are no contradictions whatsoever between the two approaches. The purpose of our paper is simply to demonstrate the increased capability of the potential formulation once a correction procedure for nonisentropic effects is included. Again, we completely agree with their comments that the entropy jump across the shock is of order $\epsilon^3\,[\epsilon = |M^2 - 1|]$, the vorticity downstream of the shock is of higher order, and so a potential exists there, at least through order ϵ^3. Moreover, only second-order terms are necessary to locate the shock wave; i.e, the entropy jump need not explicitly be found. This is obvious from comparing the exact Prandtl relationship with the small-disturbance transonic approximation (namely, the sonic point bisects the shock jump); the difference being the product of the upstream and downstream velocity perturbations, which is of order ϵ^2. Hence, one finds the same results using either the entropy jump or the Prandtl relationship.

Finally, it should be mentioned that their analytical expression for the unique incidence angle of supersonic cascades is very useful.

That having been said, it should be reemphasized that the classical potential formulation (isentropic flow) cannot be used by itself to calculate internal flows with a prescribed back pressure because, in principle, the solution may not exist unless the exit pressure happens to be the isentropic one. In this case, one has either a smooth solution or a discontinuous one, with the location of the shock wave nonunique. Our paper provides a straightforward numerical solution for this problem using a modified potential formulation. While we were not aware of the parallel asymptotic developments mentioned in the comment, we still believe the numerical approach to be not only more general but definitely simpler.

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Forty years ago in the early 1940s the advent of high-performance military aircraft that could reach transonic speeds in a dive led to a concentration of research effort, experimental and theoretical, in transonic flow. For a variety of reasons, fundamental progress was slow until the availability of large computers in the late 1960s initiated the present resurgence of interest in the topic. Since that time, prediction methods have developed rapidly and, together with the impetus given by the fuel shortage and the high cost of fuel to the evolution of energy-efficient aircraft, have led to major advances in the understanding of the physical nature of transonic flow. In spite of this growth in knowledge, no book has appeared that treats the advances of the past decade, even in the limited field of steady-state flows. A major feature of the present book is the balance in presentation between theory and numerical analyses on the one hand and the case studies of application to practical aerodynamic design problems in the aviation industry on the other.

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